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On the effect of a small counterpressure on constant-power spherical detonation waves as driven by focused radiation

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A small perturbation analysis is made of the flow inside a spherical explosion created in a gas absorbing energy from an array of lasers with a common focus. The influence of a small counterpressure outside the leading shock wave is investigated for power deposition constant in time. The departure from self-similarity is described by a power series in the variable y, the inverse square of a Mach number based on the wave-front velocity. To first order, the flow equations are solved numerically, the shock velocity is obtained and the radial profiles of velocity, density and pressure are presented graphically. The flow singularity at the radiation focus is discussed.

1. Introduction

The flow within a constant-power detonation wave created by focusing laser radiation on a gaseous target has recently been analysed by George & Moore (1973). A perturbation approach using Fourier series in the angular co-ordinate was developed under the assumption of a narrow radiation absorption layer. Self-similarity resulted when the gas was considered to be inviscid and not heat conducting and the pressure ahead of the wave front was neglected. An interesting result of that study was the appearance, in the first-order perturbation, of a strong singularity at the radiation focus. It was shown to correspond, in the fully nonlinear equations, to the existence of a 'forbidden region' into which the mathematical solution of the flow equations could not be carried.

One purpose of this paper is partially to clarify the physical understanding of this situation. To that effect, the influence of a non-zero counterpressure ahead of the leading shock wave is investigated in the case of a spherically symmetric arrangement and its bearing on the focal singularity is discussed. From another standpoint, certain experimental conditions are such that the static pressure ahead of the leading shock wave cannot be neglected in comparison with the dynamic pressure of the front.

A small perturbation analysis is made using power series in a parameter y proportional to the counterpressure. First-order velocity, density and pressure

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profiles are presented and the wave-front speed is obtained. Particular attention is paid to the behaviour of physical quantities in the neighbourhood of the radiation focus.

2. Governing equations

As in George & Moore (1973), r is the radius, t is time and the radial self-similarity variable is, for power constant in time,

$$\lambda = r/At^n \quad \text{with} \quad n = \tfrac{3}{5}, \quad A = [\pi_L \rho_0^{-1}(\gamma-1)a]^{\frac{1}{5}}, \tag{1}$$

where π_L is the laser power, ρ_0 the density of the undisturbed medium, γ the ratio of specific heats and a is a constant to be obtained as part of the solution.

Further, a non-dimensional time is introduced using the counterpressure p_0:

$$y = p_0 \rho_0^{-1} \gamma (nAt^{n-1})^{-2}. \tag{2}$$

y can be interpreted as the inverse square of a fictitious Mach number based on the sound speed ahead of the wave and wave-front speed pertaining to a self-similar spark (i.e. with zero counterpressure). This approach was found much easier to use than Sakurai's (1954) since the unknown wave-front velocity, rather than appearing in the flow equations, only enters the boundary conditions. The radial velocity, density and pressure are non-dimensionalized as

$$u_r = nAt^{n-1}V(\lambda, y), \quad \rho = \rho_0 R(\lambda, y), \quad p = \rho_0 (nAt^{n-1})^2 P(\lambda, y). \tag{3}$$

The inviscid isentropic flow equations are

$$(V-\lambda) R_\lambda + RV_\lambda + 2\frac{RV}{\lambda} - 2\frac{n-1}{n} yR_y = 0, \tag{4a}$$

$$(V-\lambda) V_\lambda + \frac{n-1}{n} V + \frac{1}{R} P_\lambda - 2\frac{n-1}{n} yV_y = 0, \tag{4b}$$

$$(V-\lambda) P_\lambda + 2\frac{n-1}{n} P + \gamma P V_\lambda + 2\gamma \frac{PV}{\lambda} - 2\frac{n-1}{n} yP_y = 0. \tag{4c}$$

Boundary conditions are applied on a sphere

$$r_d = At^n m(y), \tag{5}$$

where $m(y)$ is an *a priori* unknown function to be found as part of the solution. For convenience, define

$$\tilde{m}(y) = m(y) - 2\frac{n-1}{n} y \frac{dm(y)}{dy}. \tag{6}$$

The boundary conditions then are

$$V_d = (1-f)\tilde{m}(y), \quad R_d = 1/f, \tag{7a,b}$$

$$P_d = (1-f)\tilde{m}(y)^2 + \gamma^{-1} y \tag{7c}$$

with

$$f = \frac{1}{\gamma+1}\left\{\gamma + \frac{y}{\tilde{m}^2} - \left[\left(1 - \frac{y}{\tilde{m}^2}\right)^2 - \frac{\gamma+1}{2\pi a n^3}\frac{1}{m^2 \tilde{m}^3}\right]^{\frac{1}{2}}\right\}. \tag{7d}$$

3. Small perturbation analysis

For small counterpressures y is a small parameter. The wave-front location and flow variables can be expanded in power series in y. Retaining only the first term, one obtains

$$m(y) = 1+\lambda_2 y, \tag{8}$$

where λ_2 is an unknown constant, and

$$V = V^{(0)}(\lambda)+yV^{(1)}(\lambda), \quad R = R^{(0)}(\lambda)+yR^{(1)}(\lambda), \quad P = P^{(0)}(\lambda)+yP^{(1)}(\lambda). \tag{9}$$

The zeroth-order differential equations and boundary conditions were solved by Champetier, Couairon & Vandenboomgaerde (1968), Wilson & Turcotte (1970) and George & Moore (1973). Detailed investigation of the flow equation near the origin revealed a linear velocity profile, finite pressure and zero density, hence infinite temperature. These features are somewhat similar to those of a regular constant-energy blast wave.

The first-order flow equations are

$$R^{(0)}V^{(1)'}+\left(R^{(0)'}+2\frac{R^{(0)}}{\lambda}\right)V^{(1)}+(V^{(0)}-\lambda)R^{(1)'}+\left(V^{(0)'}+2\frac{V^{(0)}}{\lambda}-2\frac{n-1}{n}\right)R^{(1)} = 0, \tag{10a}$$

$$(V^{(0)}-\lambda)\,V^{(1)'}+\left(V^{(0)'}-\frac{n-1}{n}\right)V^{(1)}-\frac{P^{(0)'}}{R^{(0)2}}R^{(1)}+\frac{1}{R^{(0)}}P^{(1)'} = 0, \tag{10b}$$

$$\gamma P^{(0)}V^{(1)'}+\left(P^{(0)'}+2\gamma\frac{P^{(0)}}{\lambda}\right)V^{(1)}+(V^{(0)}-\lambda)P^{(1)'}+\left(\gamma V^{(0)'}+2\gamma\frac{V^{(0)}}{\lambda}\right)P^{(1)} = 0, \tag{10c}$$

where a prime denotes $d/d\lambda$. The boundary conditions are transferred to $\lambda = 1$ by a Taylor expansion:

$$V^{(1)}(1) = -K+\left[(1-f^{(0)})\frac{2-n}{n}-K_2-V^{(0)'}(1)\right]\lambda_2, \tag{11a}$$

$$R^{(1)}(1) = -\frac{K}{f^{(0)2}}-\left[\frac{K_2}{f^{(0)2}}+R^{(0)'}(1)\right]\lambda_2, \tag{11b}$$

$$P^{(1)}(1) = \frac{1}{\gamma}-K+\left[(1-f^{(0)})\,2\frac{2-n}{n}-K_2-P^{(0)'}(1)\right]\lambda_2, \tag{11c}$$

where $f^{(0)}$, K and K_2 are known constants defined as

$$f^{(0)} = \frac{1}{\gamma+1}\left[\gamma-\left(1-\frac{\gamma+1}{2\pi n^3 a}\right)^{\frac{1}{2}}\right], \tag{12a}$$

$$K = \frac{1}{4\pi n^3 a}\left(1-\frac{\gamma+1}{2\pi n^3 a}\right)^{-\frac{1}{2}}, \quad K_2 = -\frac{6-n}{n}\frac{1}{4\pi a n^3}\left(1-\frac{\gamma+1}{2\pi a n^3}\right)^{-\frac{1}{2}}. \tag{12b}$$

The system of linear differential equations (10) is numerically integrated outwards from the origin to the boundary. For this purpose a local solution of the following form is sought in the neighbourhood of $\lambda = 0$:

$$V^{(1)} \sim \lambda^x, \quad R^{(1)} \sim \lambda^{x+\alpha-3}, \quad P^{(1)} \sim \lambda^{x+\alpha-1}. \tag{13}$$

α results from the zeroth-order solution and is equal to $\frac{34}{11}$ (for $\gamma = \frac{5}{3}$). Substitution into (10) yields an indicial equation for x, the roots of which are

$$x_1 = -2, \quad x_2 = -(\alpha - 1) = -\tfrac{23}{11}, \quad x_3 = 1 + 2\frac{n-1}{n}\frac{1}{a_0 - 1} = \tfrac{31}{11}. \tag{14}$$

The first root corresponds to an $O(y)$ source of mass and energy at the origin. The physics of the problem do not allow for such an effect so the contribution from x_1 must be set equal to zero. Further, George (1972) has shown that the correct local solution corresponding to x_2 and x_3 is

$$V^{(1)}(\lambda) = B_1^1\lambda + B_1^2\lambda^{\frac{65}{11}}, \tag{15a}$$

$$R^{(1)}(\lambda) = B_2^1\lambda^{\frac{12}{11}} + A_2^2\lambda^{\frac{32}{11}}, \tag{15b}$$

$$P^{(1)}(\lambda) = A_3^1 + A_3^3\lambda^{\frac{54}{11}}, \tag{15c}$$

where B_1^1 and B_1^2 are arbitrary constants and other A's and B's are related to the former two.

The leading part of (15) is seen to have exactly the same behaviour as the local zeroth-order self-similar solution: the velocity is linear, the pressure tends to a constant and the density goes to zero like $\lambda^{\frac{12}{11}}$, hence the temperature blows up at the origin. In connexion with George & Moore's (1973) analysis of the forbidden domain around the focus of a non-spherical constant-power blast wave, it may be inferred that their physically unacceptable situation does not result from neglecting the counterpressure ahead of the wave front since in that case no singularity stronger than that in the zeroth-order solution appears.

Starting with (15), differential equations (10) are numerically integrated outwards and linearity is used to determine the unknown position λ_2 of the leading shock wave so as to satisfy boundary conditions (11). One obtains

$$\lambda_2 = -0{\cdot}1993. \tag{16}$$

This indicates a weakening of the wave owing to counterpressure. The velocity, density and pressure perturbation profiles are plotted on figure 1 versus the non-dimensional radius λ. The velocity decreases over the whole flow range whereas the density and pressure increase. Comparing the trend of this solution with Sakurai's (1954) results for constant-energy blast waves, the profiles are markedly less steep near the boundary, as might be expected from the fact that no Newtonian layer of concentrated mass exists for constant-power waves when $\gamma \to 1$.

4. Concluding remarks

Counterpressure, introduced in the form of a small perturbation, does not affect the nature of the singularity which exists at the origin of a spherical constant-power detonation wave. The locally infinite temperature still prevails and it is believed that this is the physical origin of the 'forbidden region' described by George & Moore (1973).

The present analysis gives a more refined model than the purely self-similar solution, particularly in cases where the parameter y becomes significant,

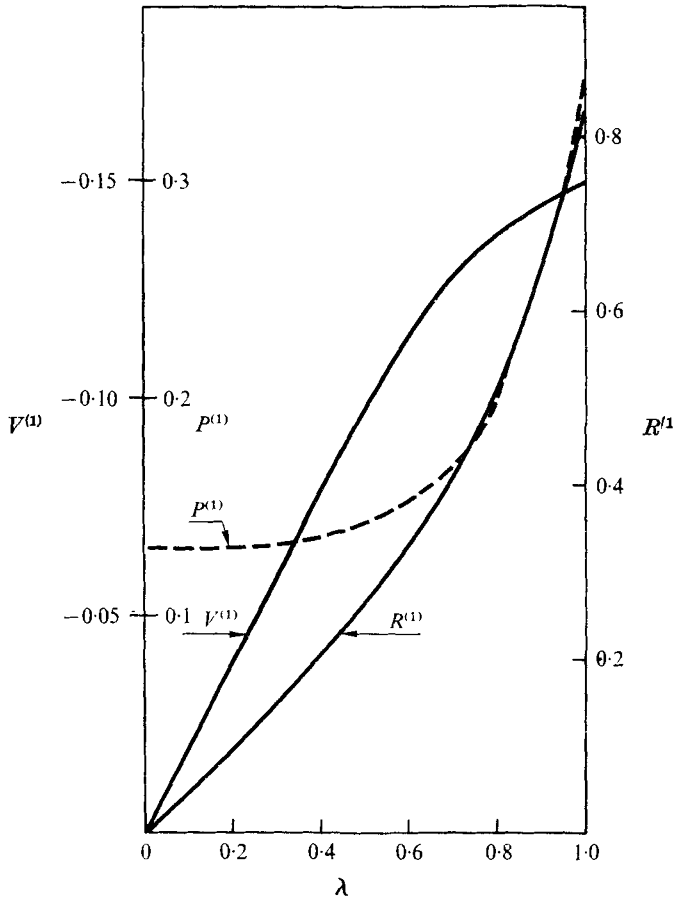

FIGURE 1. Velocity, density and pressure perturbation profiles.

i.e. when the sound speed in the undisturbed gas is not negligible compared with the wave-front speed. This was the case in an experiment by Ahmad & Key (1969) where the undisturbed gas was preheated by a primary laser detonation wave. Raizer (1968) also pointed out that the power needed to maintain a laser-supported detonation wave in air once breakdown has occurred is much smaller than the power necessary to initiate the breakdown. In that case the wave-front Mach number is no longer much larger than unity.

The technique developed could easily be applied to any power-law dependence on time of the laser power addition; the time exponent in the self-similar radial variable should then be changed. This would permit better approximation of the time dependence of the laser pulse.

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